Table Number:__________ Group Name: ________________________

Group Members:_____________________ _____________________ _____________________

Balancing Coins Part 1

Part 1: Testing Statistical Hypotheses

After talking to several expert coin balancers, we are under the impression that balancing a coin and letting it fall is an “unfair” process. In other words, this does not produce results that are 50% heads and 50% tails. *We will test the hypothesis that the proportion of heads when balancing a coin repeatedly on its edge is not 0.5.*

We will design an experiment in order to be able to make a decision about the two hypotheses. In order to make a decision about the research question, we need four things:

1. **A hypothesis to test**
2. **A sample of data which gives us a sample statistic**
3. **A sampling distribution** for the sample statistic that we obtained. This is the model we would expect if the null-hypothesis is true. We can use the sampling distribution to see how unusual or surprising our sample result is. If our sample result is in one of the tails of the distribution, it would lead us to suspect that our result is surprising given that particular model. This would therefore be evidence against the null-hypothesis.
4. **A decision rule**: How far in the tails does our sample result have to be? The decision rule tells us how far in one of the tails our sample result needs to be for us to decide it is so unusual and surprising that we reject the idea stated in the null-hypothesis.

Writing statistical hypotheses

We basically want to know if we can expect an average of 50% heads if we repeatedly balance a coin ten times, each time counting the number of heads that show or not. We can write those two ideas as follows:

Idea 1: Balancing a coin is a “fair” process.

Idea 2: Balancing a coin is an “unfair” process.

We can also write these ideas using more mathematical ideas, as shown below:

H_o : The proportion of heads when we balance a coin repeatedly is 0.5.

H_A :The proportion of heads when we balance a coin repeatedly is not 0.5.

(In other words the proportion is more, or less, than 0.5.)

We call these ideas **'statistical hypotheses'**. **The first idea** states that the coin is just a likely to land heads as it is to land tails, or that there will be an equal number of heads and tails when we balance a coin. This statement is called the **'null hypothesis'** because it represents an *idea of no difference* from the norm or prior belief or *no effect* (e.g., getting the same results as tossing fair coins). The null-hypothesis is labeled by the symbol '$\mathbf{H_0}$'.

The second idea states that there will **not** be an equal number of heads and tails, something different than the first idea, so it is called the **'alternative hypothesis'**. The symbol used for the alternative hypothesis is 'H_a'. Note that this was our original hypothesis about the proportion of heads.

We gather evidence (data) to see if we can disprove the null hypothesis. If we do, then we will accept that the alternative hypothesis is true. The decision between the two hypotheses is usually expressed in terms of H_0 (idea # 1). If our data lead us to believe the second idea is true, then we usually say that **'we reject H_0'**.

Gathering evidence (data) to make the decision whether or not to reject H_0

Balance a penny coin. Make sure the head side of the coin is facing you. When it is standing upright on its edge, hit the under-side of the table (right underneath the balanced coin). Record if you got a head or tail. Repeat this process ten times.

Trial	Result (H or T)
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

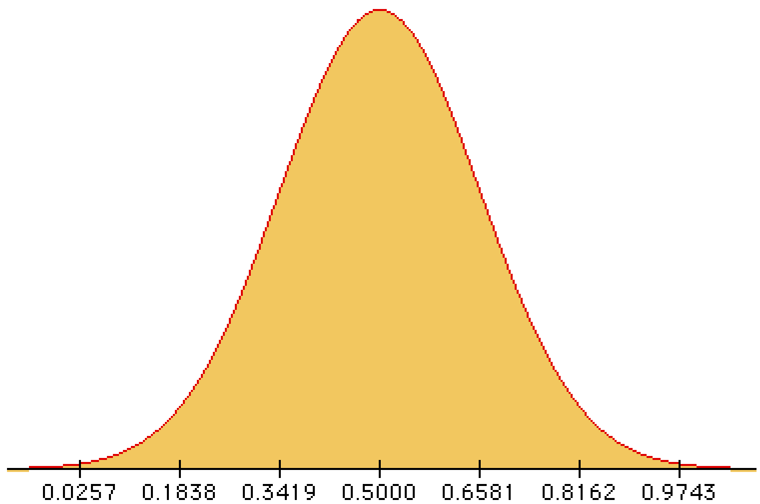
How many Heads did you get? ____

Your **sample statistic** is the *proportion of heads* (divide the number of heads by 10).

Write that here: _____

Find an appropriate sampling distribution

What shall we use? Remember that the sampling distribution is consistent with the idea expressed in the null-hypothesis. Since the null-hypothesis is that the proportion of heads is 0.5, we can refer to the sampling distribution we created earlier.



This sampling distribution allows you to compare your sample proportion of heads, based on ten trials, to other sample proportions of heads based on ten trials that you could have obtained, given that the null-hypothesis is indeed true (that balancing a coin repeatedly on its edge produces 50% heads).

Use your sketch to determine whether or not your result is in a tail. Mark your result on your graph. Does your result seem surprising? Explain.

Decision Rule

How unlikely is your sample result? To determine this, do the following:

- Open the *Stat Crunch in MyMathLab*.
- Click on STAT, then PROPORTION STATISTICS, then ONE SAMPLE, then, WITH SUMMARY.
- Enter the number of success and the number of observations.
- Enter in 0.5 after H_o . Use $\neq$ for the alternative hypothesis, H_a . This is an example of a two tail test.
- Click on Compute make note of the results, particularly the **p-value**

The **p-value** is an index of the likelihood of your sample result or a more extreme sample result under the model of the null-hypothesis.

Write your **p-value** here: _______

This probability of getting the result we got or a more extreme one is called the **p-value.** We can use the p-value to help make a decision about which of our hypotheses seems more likely. *A typical decision rule for p-values is to see if they are smaller than 0.05.* If your p-value is less than 0.05, then it would be evidence against the null hypothesis - you would *reject the null hypothesis*. If your p-value is greater than or equal to 0.05, then it would be evidence for the null hypothesis - you would *fail to reject the null hypothesis*.

How does your p-value compare to 0.05? Is it evidence *for* or *against* the null hypothesis?

What does this suggest about the "fairness" of the process of balancing a coin?

Did all your classmates come to the same conclusion? If not, why did we get different decisions in the same experiment?

What does the p-value tell us? To summarize our results we would say that the probability of getting a sample proportion such as the one we got or a more extreme sample proportion, when the null hypothesis that heads and tails are equally likely is really true is **_____.** *(Fill in the blank with the p-value.)*

References

Scheaffer, R.L., Watkins, A. Witmer, J., & Gnanadesikan, M., (2004a). *Activity-based statistics: Instructor resources* (2nd edition, Revised by Tim Erickson). Key College Publishing.

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